



Objective : Implement a fuzzy inference system for prediction of performance given input parameters based on Abrasive Water Jet Machining (AWJM) data set. Develop fuzzy rules based on dataset (expert knowledge), membership functions and parameters

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Introduction :

Ever since Lotfi A. Zadeh introduced Fuzzy Sets [1] in 1965, the field of Fuzzy Logic and Fuzzy systems has been growing rapidly with application ranging from control systems for automotive, aerospace, robotics and defense industries to decision- making support systems for business, operations, warehouse planning and scheduling. It has several applications in manufacturing, medicine, marine, transportation, pattern recognition, human behavior analysis and psychology.

Fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false.

The important components include Linguistic variables, Fuzzy variables, Membership function, Fuzzy rules, Fuzzification and Defuzzification system and Inference mechanism.

General steps for any Fuzzy Inference System (FIS) are :

- Fuzzify all input values into fuzzy membership functions.
- Execute all applicable rules in the rulebase to compute the fuzzy output functions.
- De-fuzzify the fuzzy output functions to get "crisp" output values.

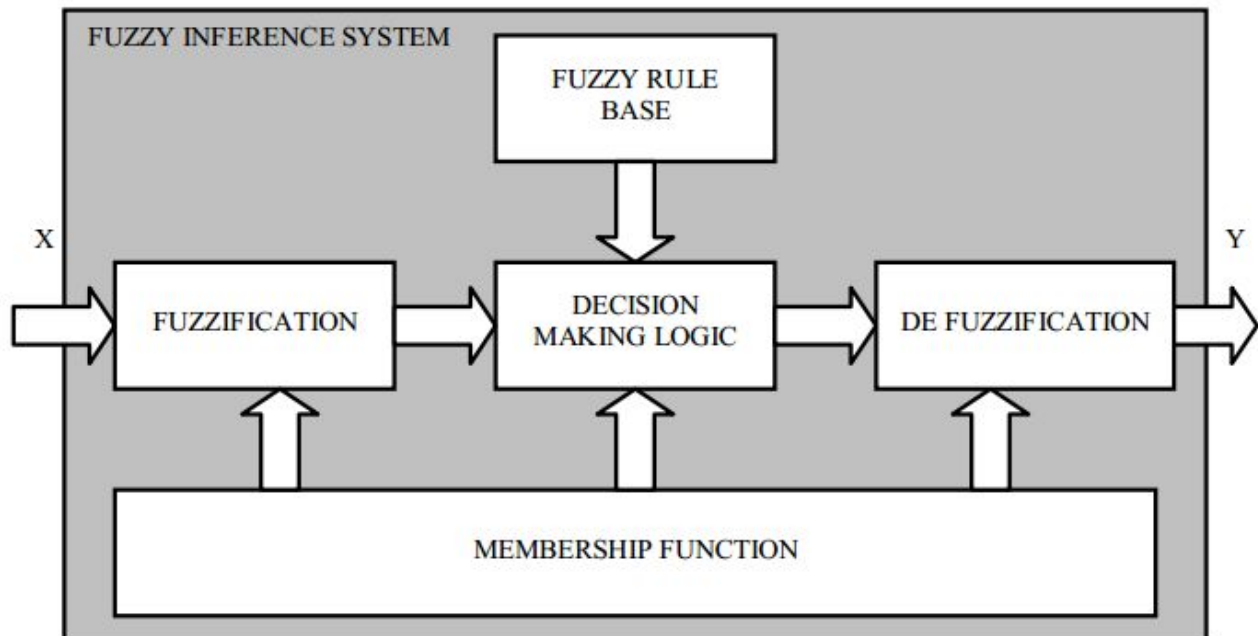


Fig. Fuzzy Logic Decision System Block Diagram



Terminology :

Important terminology in fuzzy systems are:

- **Universe of Discourse** : range of all possible values for an input to a fuzzy system
- **Fuzzy Set** : Any set that allows its members to have different grades of membership (membership function) in the interval $[0,1]$
- **Linguistic variable**: In a standard fuzzy partition, each fuzzy set corresponds to a linguistic concept, for instance Very low, Low, Average, High, Very High
- **Membership function (MF)** : Curve that defines how each point in the Universe of Discourse (input space) is mapped to a degree of membership between 0 and 1
- **Fuzzy rules** : If - Then rules than define the course of action for fuzzy logic system to reach optimal and accurate output for inputs to the system
- **Fuzzy inference** : Actual process of mapping from a given input to an output using fuzzy logic operators, membership functions and fuzzy rule base
- **Fuzzifier** : Converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base
- **Defuzzifier** : Converts the fuzzy output of the inference engine to crisp values using membership functions analogous to the ones used by the fuzzifier

Why Fuzzy Logic ?

There is a fine difference between crisp logic and fuzzy logic. Crisp logic deals with binary values of variables In contrast, fuzzy brings in a degree of human like reasoning and understanding of variables through the concept of partial truth by using values between binary limits of the variables. This can be clearly understood with an example of room temperature control of air conditioner. Studies show that the performance of room temperature control on an air conditioner has become several manifolds better and smoother while it making it much more comfortable for humans [2].

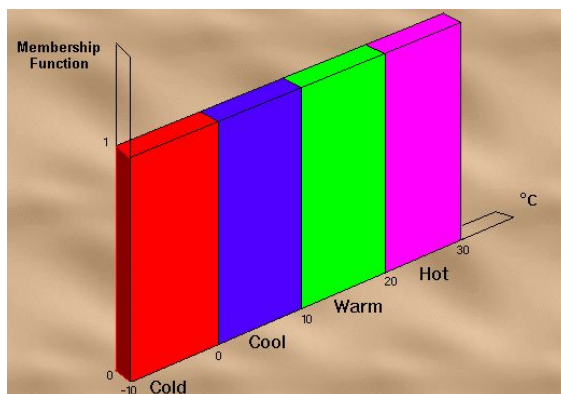


Fig. Bivalent set to characterize the room temperature

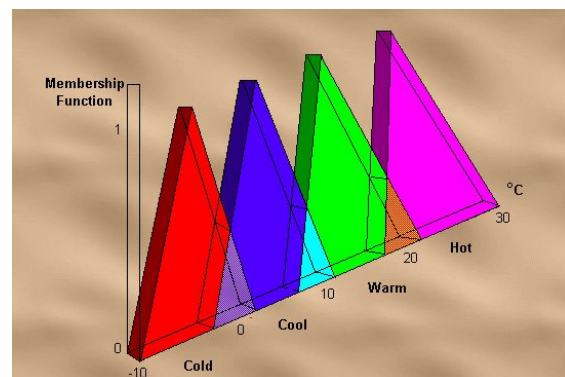


Fig. Fuzzy set to characterize the room temperature



Dataset Analysis

Through this project we try to exploit the application of FIS in control, selection and prediction of parameters for Abrasive Water Jet Machining (AWJM) process. The dataset consists of five parameters and 81 different combinations of those parameters. The parameters include Traverse Speed (TR), Abrasive flow rate (AFR), Water jet Pressure (P), Focussing Nozzle diameter (FND) in the Universe of Discourse (input space) while Depth of Cut (DOC) falls in the output space. The size of dataset is 81 X 5 with each column containing Water jet Pressure (P), Traverse Speed (TR), Abrasive flow rate (AFR), Focussing Nozzle diameter (FND), Depth of Cut (DOC)

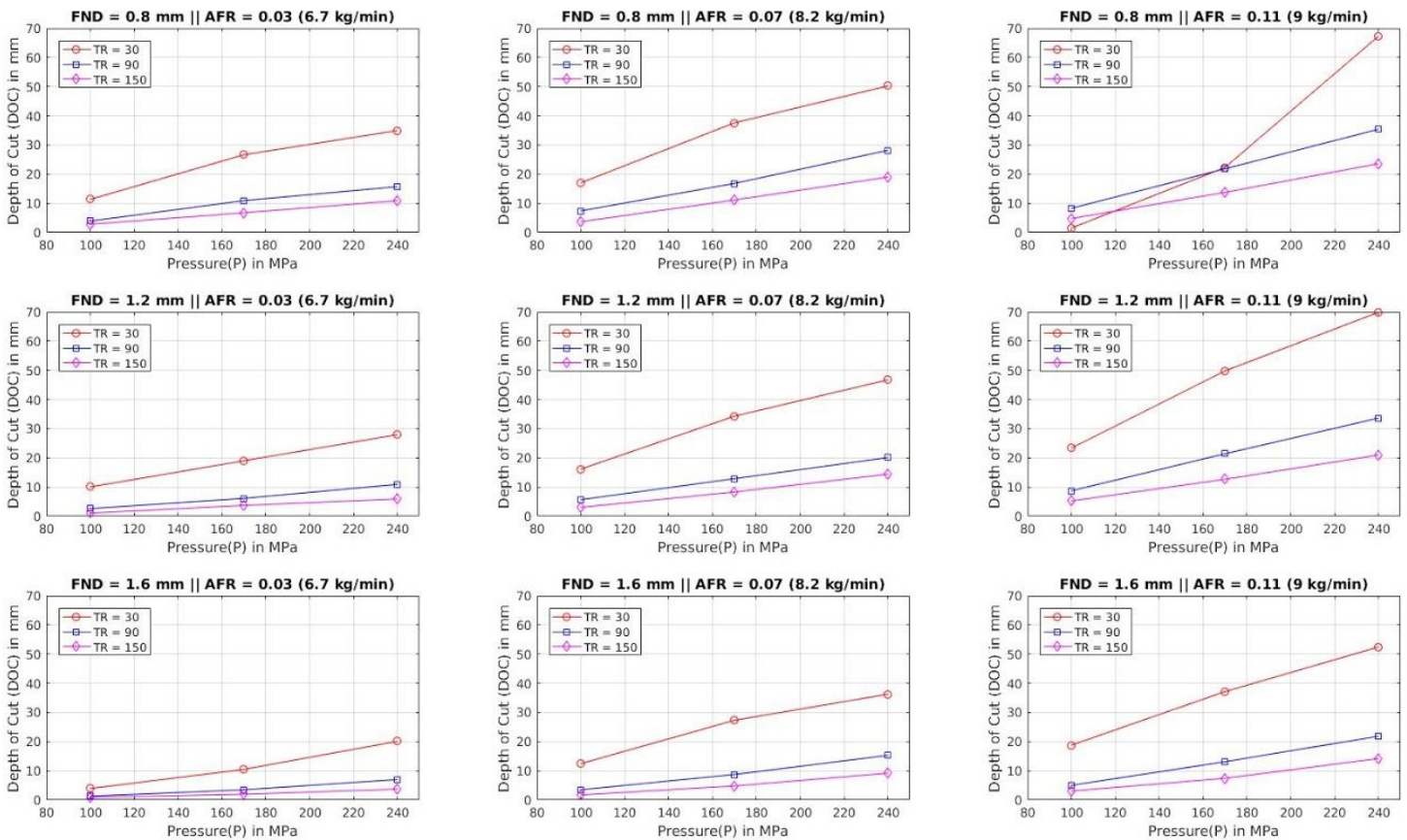


Fig. Complete dataset represented in a matrix of plots for Depth of Cut (DOC) vs Water Jet Pressure (P) with varying Focussing Nozzle diameter (FND) and Traverse Speed (TR)

Code was developed to plot the data, which has been attached in the submission .zip file Observation and approximate conclusion from the above figure are as follows :

- DOC increases in a linear fashion with P in every combination of FND, AFR and TR
- DOC decreases with TR for every case of FND and AFR
- DOC increases with AFR for constant FND



- DOC decreases with FND for constant AFR

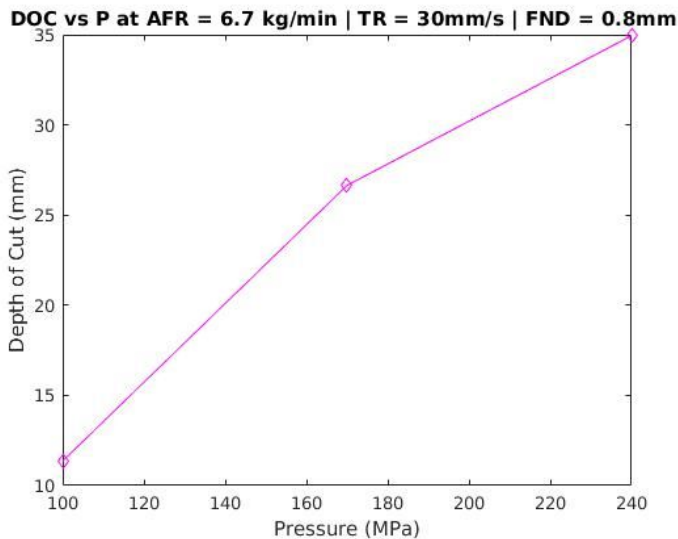


Fig. Depth of Cut increases with Pressure

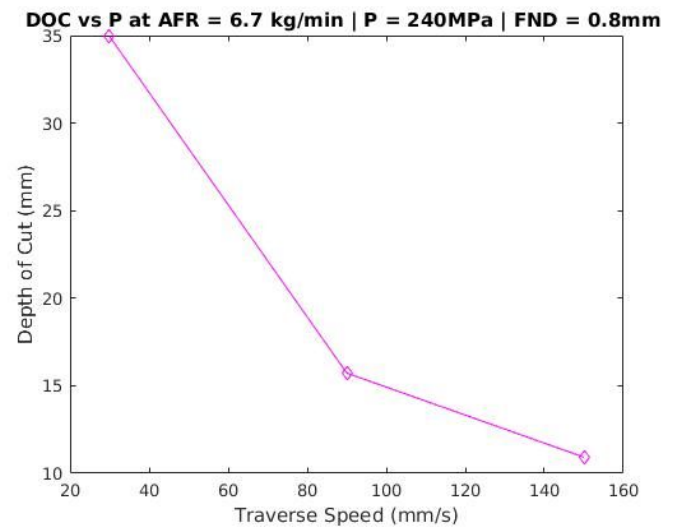


Fig. Depth of Cut decreases with Traverse Speed

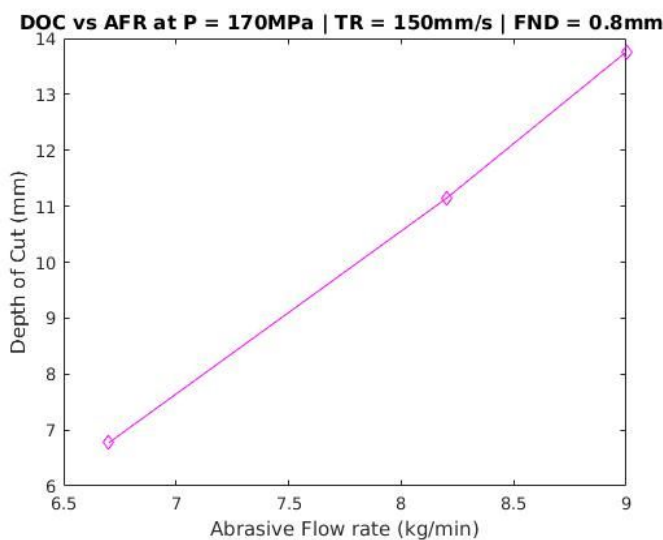


Fig. Depth of Cut increases with Abrasive Flow Rate

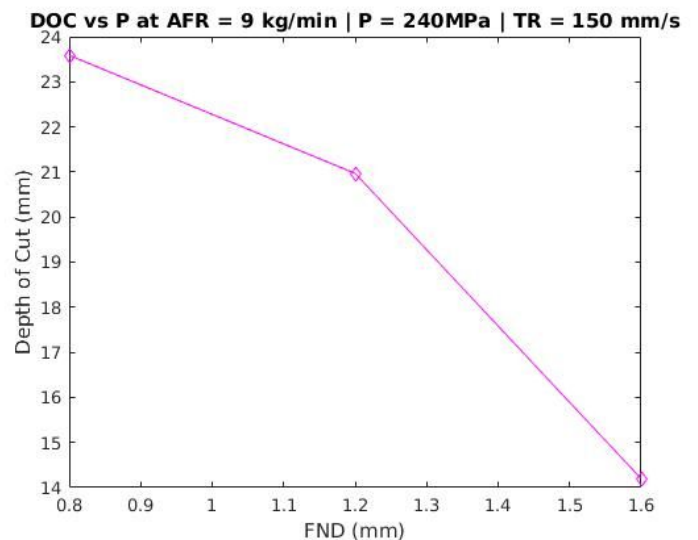


Fig. Depth of Cut decreases with Focussing Nozzle Diameter

The above figure shows individual variation of Depth of Cut with each of the four input space variables with others maintained constant. It is observed that Depth of Cut (DOC) increases with Pressure (P) and Abrasive Flow rate (AFR), while decreases with Focussing Nozzle Diameter(FND) and Traverse Speed (TR). The variation of DOC with of the input space variables can be assumed as linear.



Based on these observations the Linguistic variables and Fuzzy Rule base will be developed appropriately for prediction of Universe of Discourse variables based on values of input variables.

The inference mechanism, fuzzification and defuzzification operation will also be dependent on the above discussed trends and observations.

Defining FIS components for the given system

A system will be developed to predict Depth of cut based on value of 4 Universe of Discourse variables. Triangular membership function with three Linguistic variables classifying the Universe of discourse variables P, AFR, TR, FND into **LOW(L)**, **MEDIUM(M)**, **HIGH(H)** based on the magnitude of the value for each variable. Separate variables were defined for membership values of all the 4 input space variables against 3 linguistic variables.

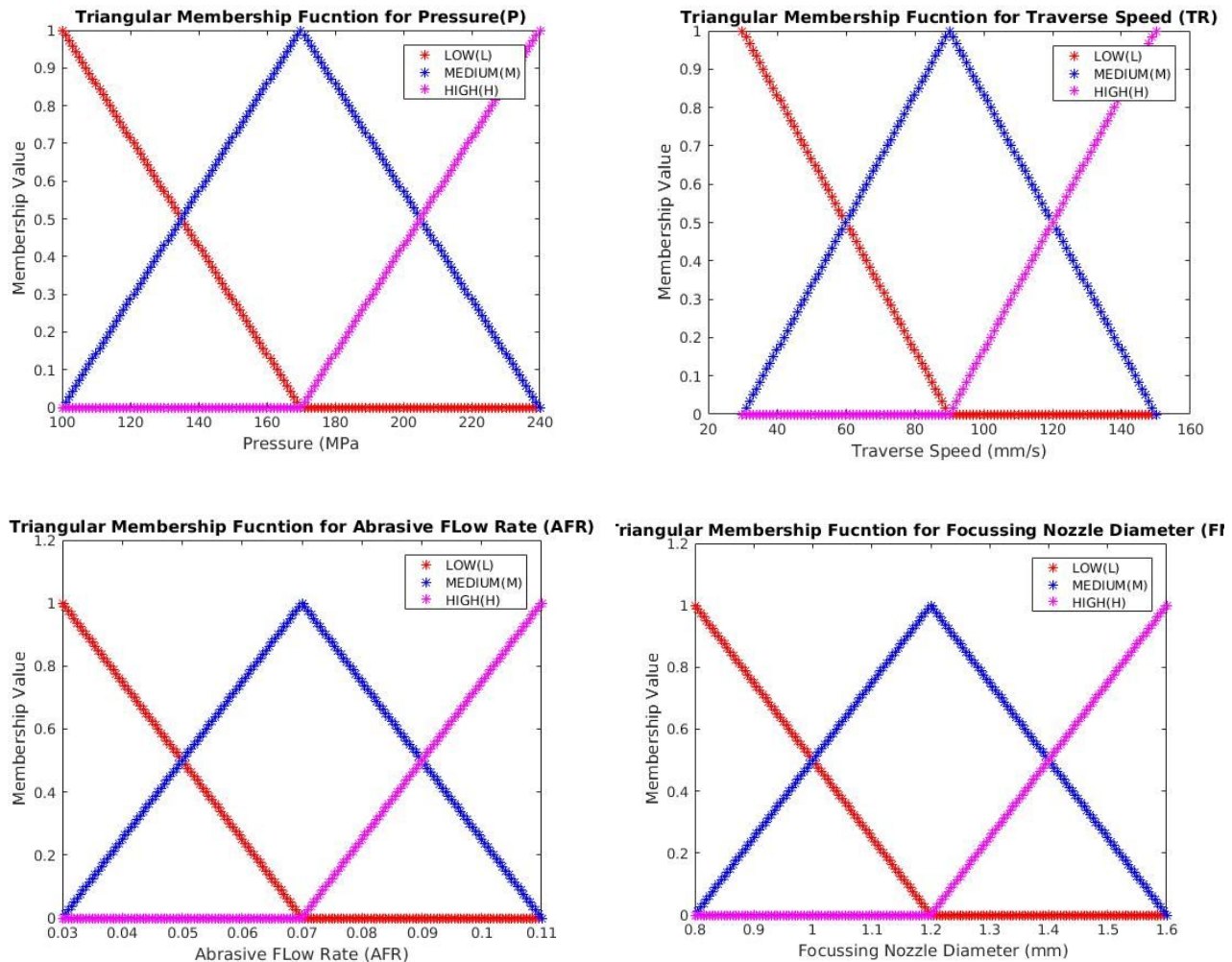


Fig. Membership Values of P, TR, AFR, FND against three linguistic variables (L, M, H)



When the data is arranged in ascending order of Depth of Cut values, we can classify the 81 sets of data into partitions based on order/magnitude of DOC. The output space variable DOC is divided into 6 linguistic variables **Very Low (VL)**, **Low (L)**, **Medium (M)**, **High (H)**, **Very High (VH)**, **Very Very High (VVH)**.

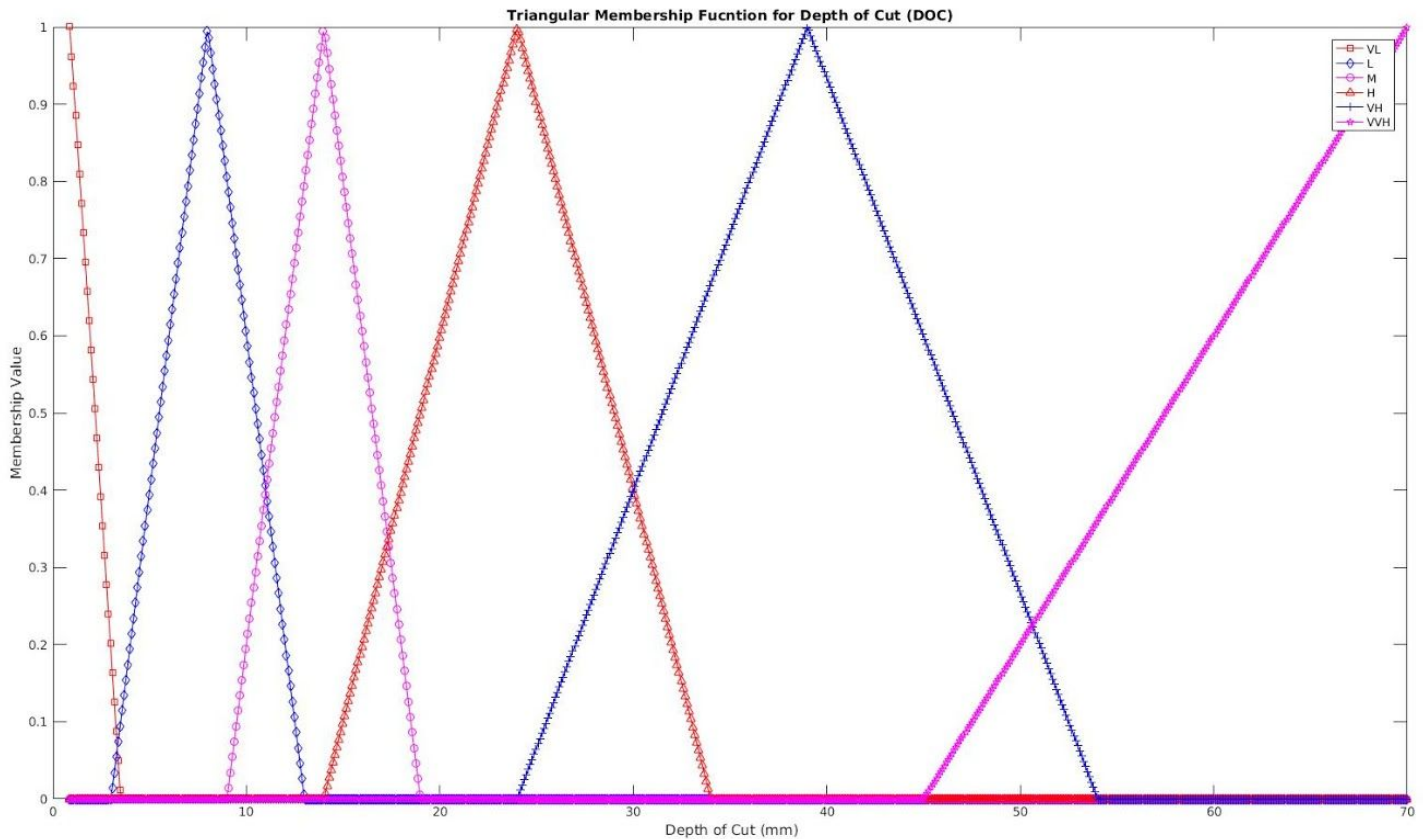


Fig. The triangular membership functions for depth of cut have chosen with irregular intervals to account for the local densities of dataset. As there are more data points at lower DOC values, shorter intervals are taken, while for higher DOC, less data points exist which drastically with change in input space variables (P, AFR, TR and FND)

The linguistic variables corresponding to value range of DOC as follows:

| Linguistic Variable | VL | L | M | H | VH | VVH |
|---------------------|-----------|--------|--------|---------|---------|---------|
| DOC (mm) | 0.87 | 8 | 14 | 24 | 39 | 70 |
| Span of variable | (0.8,3.5) | (3,13) | (9,19) | (14,34) | (24,54) | (45,70) |

The rules have been formed keeping in mind the linguistic variable ranges and comparing them with actual AWJM dataset provided.



Fuzzy Rule Development

By sorting the dataset in ascending order w.r.t DOC column, we get an actual sense of variation of DOC with input space linguistics. Since the chosen input space linguistic variables directly correspond to values in AWJM dataset, it was easy to cross check the rules. Each of the four input parameters have three linguistic variables (L, M, H). Thus a total of 3^4 (81) rules have to be devised for the data. Approximations and simplification of the rules has reduced the total number of rules defined in the code to 55 rules.

Rule : If (P is L) AND (AFR is L) AND (TR is H) AND (FND is H) **Then** DOC = VL

Code : if (P_L > 0) && (AFR_L > 0) && (TR_H > 0) && (FND_H > 0)
VL = min(P_L, AFR_L, TR_L, FND_L)

end

Rule : If (P is L) AND (AFR is L) AND (TR is H) AND (FND is H **OR** FND is M) **Then** DOC = VL

Code : if (P_L > 0) && (AFR_L > 0) && (TR_H > 0) && (FND_H > 0 || FND_M > 0)
VL = min(P_L, AFR_L, TR_L, max(FND_L, FND_M))

end

```
%Fuzzy rules

if P_L>0 && AFR_L>0 && TR_H>0 && (FND_H>0 || FND_M>0 || FND_L>0)
    VL = [VL min(P_L,AFR_L,TR_H,max(FND_H,FND_M,FND_L))];
end
if P_L>0 && AFR_L>0 && TR_H>0 && FND_H>0
    VL = [VL min(P_L,AFR_L,TR_H,FND_H)];
end
if P_L>0 && AFR_H>0 && TR_L>0 && FND_M>0
    VL = [VL min(P_L,AFR_H,TR_L,FND_M)];
end
if P_L>0 && AFR_M>0 && TR_H>0 && FND_H>0
    L = [L min(P_L,AFR_M,TR_H,FND_H)];
end
if P_H>0 && AFR_L>0 && TR_H>0 && FND_H>0
    L = [L min(P_L,AFR_L,TR_H,FND_H)];
end
if P_L>0 && AFR_L>0 && TR_M>0 && FND_M>0
    L = [L min(P_L,AFR_L,TR_M,FND_M)];
end
if P_L>0 && AFR_M>0 && TR_M>0 && (FND_M>0 || FND_L > 0)
    L = [L min(P_L,AFR_M,TR_M,max(FND_M,FND_L))];
end
```

Fig. A few examples of Fuzzy rules for producing output set using Fuzzy inference system



The general fuzzy operations are used such as:

- AND operator between L,H $\Rightarrow \min(\mu_A, \mu_B, \mu_H)$
- OR operator between L,H $\Rightarrow \max(\mu_L, \mu_H)$
- Multiplication in fuzzy sets A,B,C,D $\Rightarrow \min(\mu_A, \mu_B, \mu_C, \mu_D)$
-

After the code runs through the fuzzy rules, set of VL, L, M, H, VH, VVH are created with different membership values when called in the fuzzy if then conditional statements.

Using the Maximum Minimum reasoning scheme, actual membership of linguistic variables is the maximum of the sets of linguistic variables. So $\mu_{VL}(\text{final}) = \{\max(\mu_{VL}(x_i)) \mid \text{for all } x_i\}$.

Max-Min Composition is used.

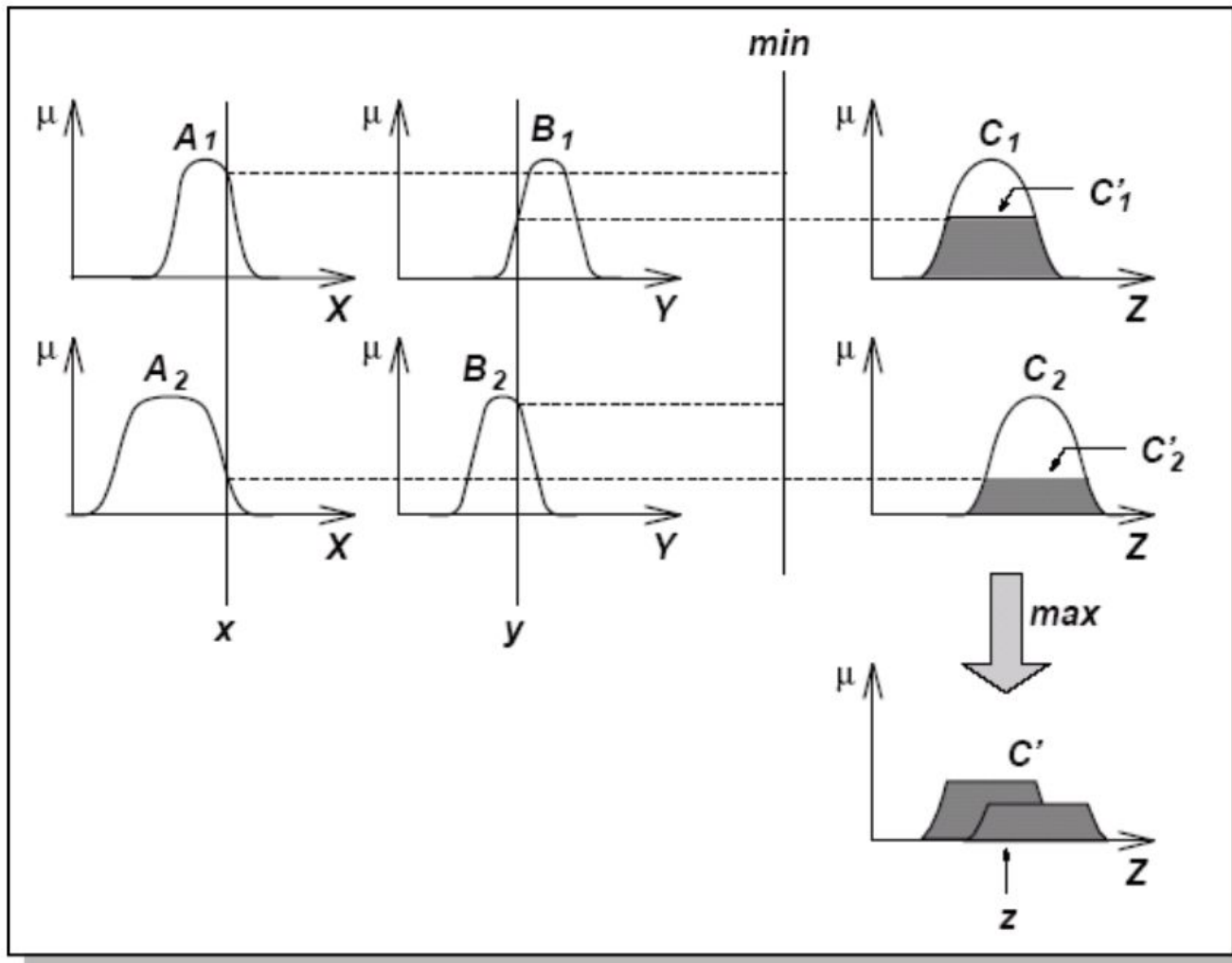


Fig. The Max - Min reasoning scheme working for a set of final membership values

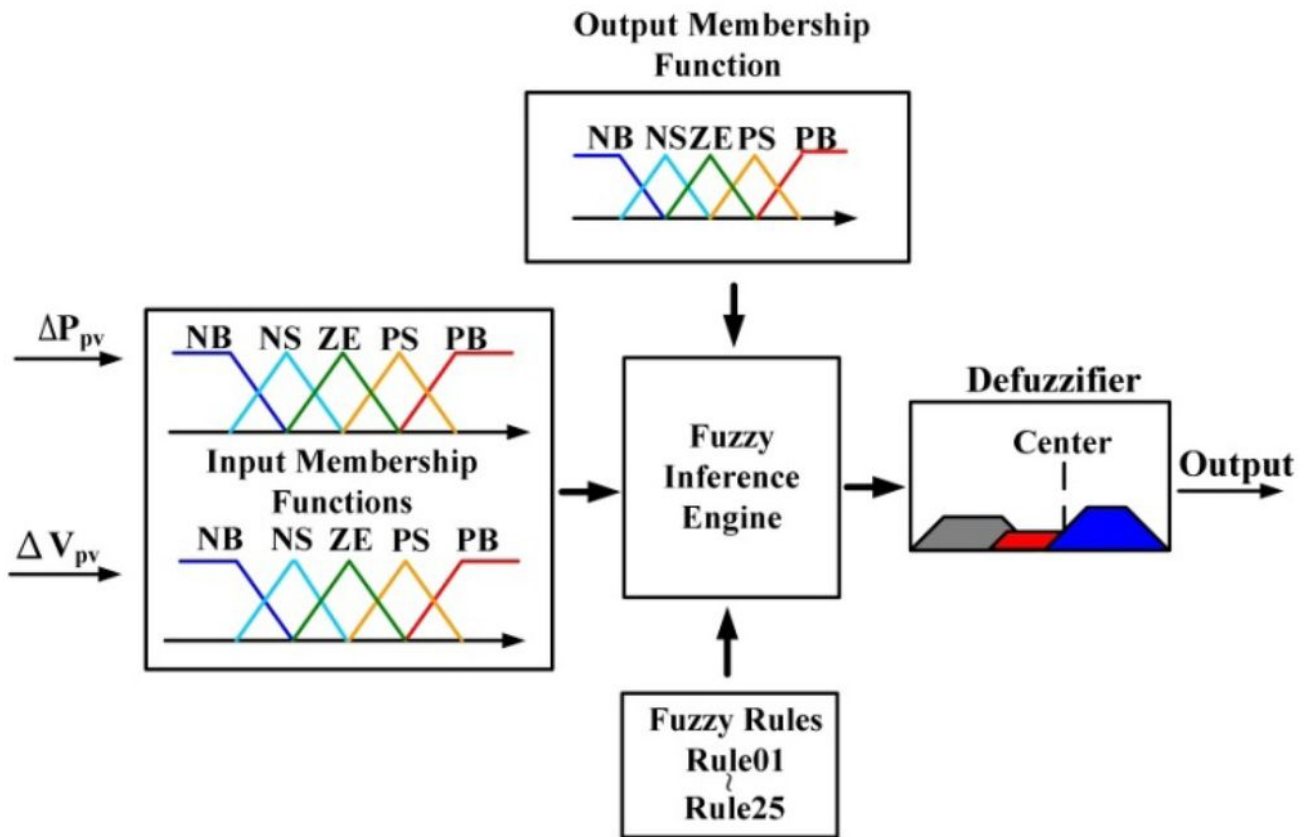


Fig. Fuzzy Inference System diagram depicting the essential components

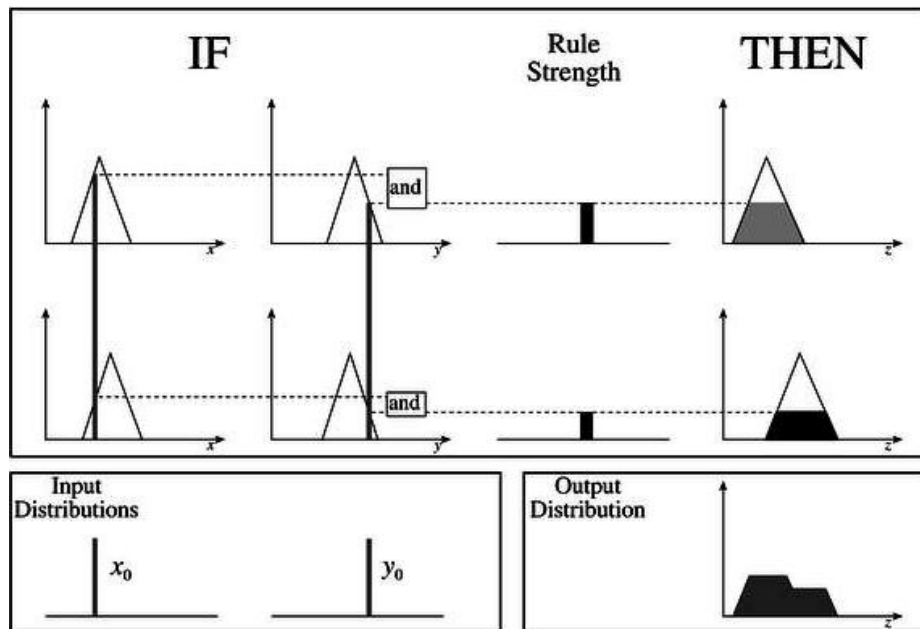


Fig. Figure depicting the working mechanism of fuzzy inferencing with rule strength
Defuzzification



After computing maximum of the membership sets for each of 6 output space linguistic variable, final membership for each of linguistic variable is obtained based on maximum - minimum reasoning scheme. **Centroid method (Centre of Areas)** is used to defuzzify the fuzzy output set into a crisp value of Depth of Cut (CrispDOC). Each membership triangle is truncated into a trapezium at the height of its membership value(Y-axis).Abscissa of Centre of the local triangular membership area(x_i) is equal to value assigned to linguistic variable (by definition) with a membership value equal to μ_i . Let del_i be base length of the triangular area which is equal to difference in the upper and lower limits of span of linguistic variables.

Base length of Trapezium = del_i

Length of side opposite to base = $del_i * (1 - \mu_i)$

Height of Trapezium = μ_i

Using the definition of area of Trapezium:

Area of trapezium (A_i) = $(1/2) * (\text{Base length} + \text{Length of opposite side}) * (\text{Height of Trapezium})$

Area of trapezium (A_i) = $(1/2) del_i^2 \mu_i (1 - \mu_i)$

CrispDOC = $\sum_i (\text{Area of Trapezium} * x_i) / \sum_i (\text{Area of Trapezium})$

CrispDOC = $\sum_i (A_i * x_i) / \sum_i (A_i)$ where $i = VL, L, M, H, VH, VVH$

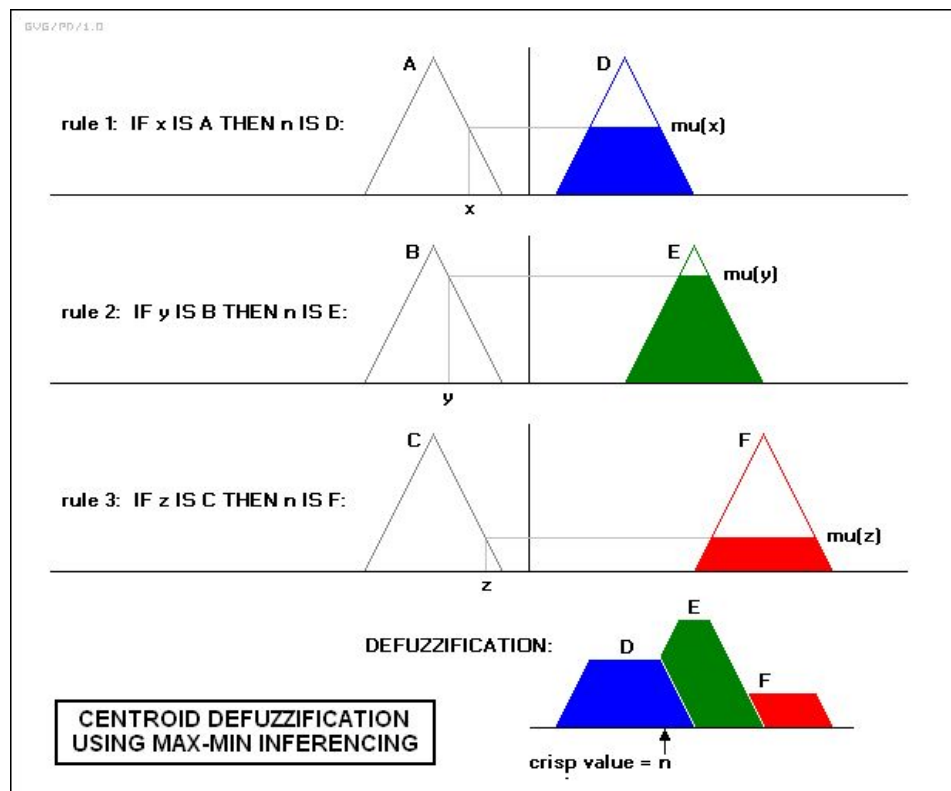


Fig. Geometrical representation of Centroid Defuzzification Technique



Results

An executable code has been developed to predict Depth of Cut by taking in textual inputs of input parameters (Focussing Nozzle Diameter, Abrasive Flow rate, Traverse Speed, Water Jet Pressure) through command window of MATLAB. The results for prediction accuracy have provide in the following table :

| Predicted DOC | 14mm | 8mm | 34.384mm | 52.04mm |
|------------------|---------|--------|----------|---------|
| Actual DOC | 12.66mm | 7.42mm | 27.95mm | 46.44mm |
| Error Percentage | 9.57% | 7.25% | 18.71% | 10.76% |

Conclusion

A fuzzy Inference system has been developed for predicting Depth of cut for Abrasive Water Jet Machining process when provided with input parameters - Focussing Nozzle Diameter, Abrasive Flow rate, Traverse Speed, Water Jet Pressure. MATLAB executable code has been developed **without usage of any Fuzzy Logic Tool Boxes** or any other Add ons from MATLAB. A general framework was developed for Inference Mechanism and Fuzzy Rule Base, Fuzzification, Defuzzification which can be expanded to other applications with ease.

Table of References

| Reference Number | Title of the Paper/Journal | Name of the Journal/ Publication/Conference | Year |
|------------------|--|---|----------|
| 1 | L.A. Zadeh - Fuzzy Sets | NA | 1965 |
| 2 | Toshikazu Tobi - "A practical application of fuzzy control for an air-conditioning system" | International Journal of Approximate Reasoning VOL. 5 | May,1991 |
| 3 | Jean J. Saade and Hassan B. Diab - "Defuzzification Methods and New Techniques for Fuzzy Controllers" | IRANIAN JOURNAL OF ELECTRICAL AND COMPUTER ENGINEERING - VOL. 3 Summer Fall | 2014 |
| 4 | Kovacevic, Radovan, and Mei Fang. - "Modeling of the influence of the abrasive waterjet cutting parameters on the depth of cut based on fuzzy rules" | International Journal of Machine Tools and Manufacture 34.1 (55 -72) | 1994 |
| 5 | Sitarama Chakravarthy, P., and N. Ramesh Babu. - "A new approach for selection of optimal process parameters in abrasive water jet cutting" | Materials and manufacturing processes 14.4 (1999): 581-600 | 1999 |